

On Meteoroid Streams Identification

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Abstract. Criterion for the membership of individual meteors to meteoroid streams presented by Valsecchi *et. al.* (1999) and Jopek *et. al.* (1999) is discussed. The authors characterize and use their criterion as a distance function. However, it is not a distance function. Some practical aspects are also discussed. Correct criterion is presented.

Key words: interplanetary medium - meteoroids – meteor streams

1. Introduction

Valsecchi *et. al.* (1999) introduce a new approach to meteoroid streams identification, based on a distance function involving four quantities. Application of the new criterion is presented by the same authors in Jopek *et. al.* (1999).

The authors have formulated the new criterion as follows:

$$[D_N]^2 = (U_2 - U_1)^2 + w_1 (\cos \vartheta_2 - \cos \vartheta_1)^2 + (\Delta \xi)^2, \quad (1)$$

$$(\Delta \xi)^2 = \min \{ w_2 \Delta \phi_I^2 + w_3 \Delta \lambda_I^2, w_2 \Delta \phi_{II}^2 + w_3 \Delta \lambda_{II}^2 \}, \quad (2)$$

$$\Delta \phi_I = 2 \sin \left(\frac{\phi_2 - \phi_1}{2} \right), \quad (3)$$

$$\Delta \phi_{II} = 2 \sin \left(\frac{180^\circ + \phi_2 - \phi_1}{2} \right), \quad (4)$$

$$\Delta \lambda_I = 2 \sin \left(\frac{\lambda_2 - \lambda_1}{2} \right), \quad (5)$$

$$\Delta \lambda_{II} = 2 \sin \left(\frac{180^\circ + \lambda_2 - \lambda_1}{2} \right) \quad (6)$$

and w_1 , w_2 and w_3 are “suitably” defined weighting factors, see Eqs. (23) – (28) in Valsecchi *et. al.* (1999). The relation between the quantity U and the well known Tisserand parameter T is given by the relation

$$U = \sqrt{3 - T},$$

$$T = \frac{1}{a} + 2 \sqrt{a (1 - e^2)} \cos i, \quad (7)$$

where the semimajor axis a is measured in AU (e is eccentricity, i is inclination), see Eqs. (8) and (9) in Valsecchi *et. al.* (1999). The quantity $\cos \vartheta$ is given by the formula

$$\cos \vartheta = \frac{1 - U^2 - 1/a}{2 U} \quad (8)$$

(see Eq. (21) in Valsecchi *et. al.* (1999). The quantity λ is a longitude of a meteoroid. The angle ϕ is defined by Eqs. (10) – (12) in Valsecchi *et. al.* (1999):

$$U_x = U \sin \vartheta \sin \phi,$$

$$U_y = U \cos \vartheta,$$

$$U_z = U \sin \vartheta \cos \phi, \quad (9)$$

where U_x , U_y and U_z are components of geocentric encounter velocity \mathbf{U} . The values of the three weighting factors (w -factors) are equal to 1 in Jopek *et. al.* (1999).

2. Mathematics and the new D-criterion

Valsecchi *et. al.* (1999) were inspired by the D-criterion of Southworth and Hawkins (1963). Thus, the authors were inspired by the well-known definition of the distance in Euclidean space. As a consequence they have obtained new D-criterion in the form of Eqs. (1) – (6). As it is considered, D-criterion measures the distance between orbits of two meteoroids. However, if it is so, then the new D-criterion (1) must fulfill properties required for a quantity called distance. The standard properties of a distance are closely connected with the so-called metric space. Definition states:

Let X be a set with elements u, v, w, \dots . A nonnegative function ρ defined on the Cartesian product $X \times X$ is called a *metric* if it satisfies the following axioms:

- (i) $\rho(u, v) = 0$ if and only if $u = v$;
- (ii) $\rho(u, v) = \rho(v, u)$;
- (iii) $\rho(u, v) \leq \rho(u, w) + \rho(w, v)$.

A set X with a metric ρ is called a *metric space*.

(Metric – distance. The property (iii) is called *the triangle inequality*.)

Now, question is, if these properties are fulfilled also for D-criterion (1). One can easily verify that triangle inequality is violated. It means that triangle inequality, which is an evident property of a distance, is not fulfilled in the case of measuring “distances” between meteoroid orbits.

As an evident property of the D-criterion we introduce the following one. If $D(u, v)$ is smaller than $D(u, w)$, then orbits u and v are more similar than the orbits u and w . However, due to the violation of the triangle inequality, the orbits v and w may be more similar than one would expect on the basis of general conception about distance:

$$0 < D(v, w) < D(u, w) - D(u, v) .$$

We can formulate this in terms of meteoroid orbits: the distance between meteoroids u and v is small, the distance between meteoroids u and w is large, but the distance between meteoroids v and w may be small.

One must work with a D-criterion which fulfills the properties (i)-(iii), from the mathematical point of view.

3. New D-criterion and its application

One must be aware of some other facts, applying the criterion defined by Eqs. (1) – (6). We show two examples.

1. Let us consider σ Leonids presented in Table 2 (p. 270) in Southworth and Hawkins (1963). The new criterion yields that the object “53 March 19.39518” belongs to the stream although its orbital parameters are $a = 8.49$ AU and $e = 0.913$ (all the other objects have $a \in (1, 3)$ AU, $e < 0.8$). Then reason is evident: $a(1 - e^2)$ compensates the extremes in a and e , and, $1/a$ is small in Eqs. (7) and (8). Thus, initial election (from a large set of data) of possible candidates to a meteoroid stream must take into account this unpleasant property.

2. Let us consider Taurid meteoroid complex. The result of Jopek *et. al.* (1999) states that the number of the members, classified with the new D-criterion, is in $\approx 10\%$ greater than the number of members classified with the D-criterion of Southworth and Hawkins. However, such a consistency says nothing about the quality of the both criteria. Really, if we take into account the Lund database and take a rough criterion, based only on distributions in π and i ($\pi \in (110^\circ, 190^\circ)$; $i < 8.5^\circ$), we obtain that the number of the obtained members is in $\approx 30\%$ greater than the number of members classified with the D-criterion of Southworth and Hawkins. This number will, of course, decrease when

other two variables (orbital elements) are used: e. g., the use of semimajor axis ($1/a \in (0.26, 0.74)$) reduces the number 30 % to ≈ 17 %, and, the following reduction due to the distribution in eccentricity ($e \in (0.78, 0.90)$) reduces the number ≈ 17 % to ≈ 1 % (comparison with Porubčan and Štohl, 1987).

The consequence of the point 2 is that the used D-criteria are useless complications. It is caused by incorrect methods, and, also (mainly) by the present state of small number of precise orbits.

Another problem may seem to be important. Let us take a meteoroid stream. The question is, if all the individual terms of the sum in a given distance $D(A, B)$ should be comparable. If this is the only requirement for the choice of the criterion for practical usage, then we can write criterion at once:

$$D_N = |U_2 - U_1| + w_1 |\cos \vartheta_2 - \cos \vartheta_1| + \Delta\xi, \quad (10)$$

where

$$\Delta\xi = \min \{w_2 |\Delta\phi_I| + w_3 |\Delta\lambda_I|, w_2 |\Delta\phi_{II}| + w_3 |\Delta\lambda_{II}|\}, \quad (11)$$

and, Eqs. (3) – (6) hold. The choice of the (positive) weighting factors is made in the way that all the four members in D_N are comparable. The criterion defined by Eqs. (10), (11) and (3) – (6) has also one important property: it fulfills the triangle inequality.

4. Correct Access

Since there does not exist any physics which can define, in a simple way, a meteoroid stream, we take the data (set of quantities for various meteoroids) as a random sample. The method was described in Klačka (1995):

If we define the meteor stream as a set of bodies with elements $X \in \Omega$,

$$P(X \in \Omega) \equiv \int_{\Omega} f(X) dX = \alpha, \quad (12)$$

(f is density function) then there is a probability less than $1 - \alpha$ that objects with $X \in \Omega'$ belong to the stream. The area Ω may be taken in various (infinity) ways. We may take even the area $\Omega : D_N \leq D_c$, where D_N is defined by Eqs. (1) – (6) (or Eqs. (10) – (11)), if one of the indices 1 and 2 is fixed (say, it represents mean values of the quantities $U, \cos \vartheta, \phi, \lambda$).

The method just described also explains why the “**distance**” method (comparing distances between individual meteoroids) is **incorrect**. The text beneath the point 2 in the preceding section is now easily understandable, also. Only a small number of objects

is known and their corresponding points in the corresponding phase space (of orbital elements or other quantities) are far from a continuous set.

The method just described also explains why various authors may obtain different results for classification of a meteoroid stream: they consider different areas Ω . If we define the stream through a density function corresponding to multidimensional normal distribution, we can take areas Ω as dispersion ellipsoids (of course, other possibilities exist): this may be important in the case when two quantities are highly correlated (e.g., the quantities U and $\cos \vartheta$ yields $r > 0.8$ for correlation coefficient in the example 1 in section 3).

5. Conclusion

We have shown that method suggested by Valsecchi *et. al.* (1999), and used by Jopek *et. al.* (1999), is not correct. We have stressed some aspects which should be taken into account when making a classification of meteoroid streams. We have presented correct method for determining a meteoroid stream, based on probability theory and statistical mathematics.

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